

A prescription for the turbulent heating of astrophysical plasmas

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ABSTRACT

The ratio of ion to electron heating due to the dissipation of Alfvénic turbulence in astrophysical plasmas is calculated based on a cascade model for turbulence in weakly collisional plasmas. Conditions for validity of this model are discussed, a prescription for the turbulent heating is presented, and it is applied to predict turbulent heating in accretion disks and the interstellar medium.

Key words: turbulence: plasma – heating

1 INTRODUCTION

Plasma is a ubiquitous form of matter in the universe, nearly always found to be magnetized and turbulent. In a wide variety of space and astrophysical plasmas, the dissipation of turbulent fluctuations makes a significant contribution to the thermodynamic heating of the plasma, requiring an understanding of this behavior to interpret a large body of astronomical observations. Recently, there has been much vigorous activity in the study of turbulent heating in a wide variety of systems, including galaxy clusters (Chandran 2004; Dennis & Chandran 2005), the interstellar medium (Minter & Spangler 1997; Struck & Smith 1999; Spangler 2003; Scalo & Elmegreen 2004; Lerche et al. 2007), accretion disks (Balbus et al. 1994; Quataert 1998; Gruzinov 1998; Quataert & Gruzinov 1999; Sharma et al. 2007; Rossi et al. 2008), molecular clouds (Pan & Padoan 2009; Lemaster & Stone 2009), the solar corona (Velli 2003; Verdini et al. 2009; Chandran 2010), the solar wind (Saito et al. 2008; Valentini et al. 2008; Breech et al. 2009; Cranmer et al. 2009; Lehe et al. 2009; Chandran et al. 2010), and the magnetospheres of the Earth (Chaston et al. 2008) and of other planets (Saur 2004).

For many of these plasma environments, the dissipation of turbulent fluctuations occurs at length scales, in the direction parallel to the local mean magnetic field, that are small compared to the particle mean free paths. The resulting weakly collisional dynamics then requires a kinetic description (Howes et al. 2008b,c), with the dissipation occurring via collisionless wave-particle interactions (Howes 2008; Schekochihin et al. 2009). These kinetic damping mechanisms generally transfer energy to the plasma ions and electrons at different rates. The time scale for the collisional

equilibration of temperature between species is often long, allowing the plasma ions and electrons to maintain distinct heating rates and temperatures. The radiation emitted from the hot, magnetized plasma will often depend strongly on the nature of the turbulent plasma heating, exerting a dominant influence on the observational signature of the object.

Until very recently, the question of differential heating of the ions and electrons by plasma turbulence had not been widely considered, with the exception of a pioneering series of papers by Quataert and Gruzinov (Quataert 1998; Gruzinov 1998; Quataert & Gruzinov 1999). The need for an accurate description of the turbulent plasma heating by kinetic mechanisms motivates the two primary aims of this Letter: (1) to calculate the relative heating of ions and electrons due to the kinetic dissipation of plasma turbulence; and (2) to provide a prescription for turbulent plasma heating that may be used as a sub-grid-scale model for heating in simulations of astrophysical plasma turbulence.

2 TURBULENT HEATING MODEL

The turbulent heating prescription determined in this Letter is based on the Howes et al. (2008a) model for the turbulent cascade of energy in a weakly collisional plasma. Whereas that study focused on conditions relevant to turbulence in the near-Earth solar wind, here we generalize to a more broad range of plasma parameters for general astrophysical applications. A brief description of the model and its underlying assumptions follows—more detailed discussion is found in Howes et al. (2008a).

The cascade model makes three primary assumptions: (1) the Kolmogorov hypothesis that the energy cascade is determined by local interactions (Kolmogorov 1941); (2) the turbulence maintains a state of critical balance at all scales (Goldreich & Sridhar 1995); and (3) the linear kinetic damp-

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ing rates are applicable in the nonlinearly turbulent plasma. Modern theories of MHD turbulence (Goldreich & Sridhar 1995; Boldyrev 2006) predict an anisotropic cascade of turbulent energy through wave vector space in a magnetized plasma¹. The resulting state of critical balance is supported by numerical simulations (Cho & Vishniac 2000) and observations in the solar wind (Horbury et al. 2008; Podesta 2009). A recent study of test particle heating in MHD turbulence supports the use of linear theory to estimate the damping and heating in a turbulent plasma (Lehe et al. 2009).

Astrophysical turbulence typically supports a large inertial range at perpendicular scales larger than the ion Larmor radius $k_\perp \rho_i < 1$, leading to turbulent fluctuations at $k_\perp \rho_i \gtrsim 1$ that are highly anisotropic with $k_\parallel \ll k_\perp$. Such anisotropic fluctuations are optimally described by a low-frequency expansion of kinetic theory called gyrokinetics (Rutherford & Frieman 1968; Frieman & Chen 1982; Howes et al. 2006; Schekochihin et al. 2009). In this limit, all collisionless dissipation occurs via the Landau resonance, and the cyclotron resonance is negligible. Limitations of the application of gyrokinetic theory in the solar wind are discussed in detail in Howes et al. (2008a). Energy spectra from the first nonlinear gyrokinetic simulations of Alfvénic turbulence at the scale of ion Larmor radius are modeled with striking agreement by the cascade model used in this Letter (Howes et al. 2008c).

A key simplification of the physics in the gyrokinetic limit is that the normalized eigenfrequency of the linear, collisionless gyrokinetic dispersion relation (Howes et al. 2006) is independent of k_\parallel , giving $\varpi(k_\perp, \beta_i, T_i/T_e) = \omega(k_\perp, k_\parallel, \beta_i, T_i/T_e)/k_\parallel v_A$; the same holds for the damping rate, $\bar{\gamma} = \gamma/k_\parallel v_A$. This enables the development of a one-dimensional model of the turbulent magnetic field energy (in velocity units), $b_k^2 \equiv \delta B_\perp^2(k_\perp)/4\pi n_i m_i$, where $\delta B_\perp^2(k_\perp)/8\pi$ is the energy density of the magnetic field fluctuations perpendicular to the mean field integrated over k_\parallel .

The model assumes a fully ionized plasma of protons and electrons with Maxwellian equilibrium velocity distributions of temperature T_s (in energy units). Plasma thermal velocities are taken to be non-relativistic, $v_{ts} = \sqrt{2T_s/m_s} \ll c$. At the scales of dissipation (of order the ion Larmor radius), the mean magnetic field can be modeled as a straight, uniform field of magnitude B_0 . The turbulence is assumed to be sub-Alfvénic, driven isotropically at a wave number k_0 with an amplitude satisfying the critical balance condition, and balanced (equal Alfvén wave energy fluxes in opposite directions along the magnetic field). The model follows the magnetic energy in the Alfvénic cascade as it transitions from Alfvén waves to kinetic Alfvén waves at $k_\perp \rho_i \sim 1$ (Howes et al. 2008c; Howes 2008; Schekochihin et al. 2009). The numerical algorithms used to calculate the linear gyrokinetic and Vlasov-Maxwell dispersion relations are described in Quataert (1998) and Howes et al. (2006).

The continuity equation for the evolution of the magnetic energy versus perpendicular wavenumber is given by

$$\frac{\partial b_k^2}{\partial t} = -k_\perp \frac{\partial \epsilon(k_\perp)}{\partial k_\perp} + S(k_\perp) - 2\gamma b_k^2, \quad (1)$$

¹ Slightly different predictions for inertial range spectral slopes $k_\perp^{-5/3}$ or $k_\perp^{-3/2}$ are unlikely to yield significant differences in the turbulent heating ratios predicted by this model.

where the energy cascade rate is $\epsilon(k_\perp) = C_1^{-3/2} k_\perp \bar{\omega} b_k^3$, the energy injection rate is S (non-zero only at the driving scale $k_\perp = k_0$), and the linear kinetic damping rate is γ . Assuming critical balance at all scales, the steady state solution for the energy cascade rate is given by

$$\epsilon(k_\perp) = \epsilon_0 \exp \left\{ - \int_{k_0}^{k_\perp} 2C_1^{3/2} C_2 \frac{\bar{\gamma}(k'_\perp)}{\bar{\omega}(k'_\perp)} \frac{dk'_\perp}{k'_\perp} \right\}, \quad (2)$$

where C_1 and C_2 are order unity dimensionless Kolmogorov constants and ϵ_0 is the rate of energy input at k_0 . The solution of $\epsilon(k_\perp)$ for parameters $(\beta_i, T_i/T_e)$ is then used, along with the linear gyrokinetic damping rates due to each species $\bar{\gamma}_s$ (Howes et al. 2006), to calculate the spectrum of heating by species $Q_s(k_\perp) = 2C_1^{3/2} C_2 (\bar{\gamma}_s/\bar{\omega}) \epsilon(k_\perp)/k_\perp$. After integrating over k_\perp , we obtain the total ion-to-electron heating rate due to the kinetic dissipation of the turbulent cascade, $Q_i/Q_e(\beta_i, T_i/T_e)$, the primary scientific result of this Letter. Note that Quataert & Gruzinov (1999) presented a similar model to estimate the fraction of electron heating for $T_i/T_e = 100$ and $1 \leq \beta_i \leq 10^3$, but did not account for the transition to a kinetic Alfvén wave cascade at the scale of the ion Larmor radius $k_\perp \rho_i \sim 1$.

2.1 Limitations of the Model

First, the cascade at weakly collisional scales with $k_\perp \rho_i < 1$ is assumed to consist entirely of Alfvénic fluctuations with Alfvén Mach number less than unity. The energy in compressive modes, such as the collisionless manifestation of the MHD fast and slow magnetosonic waves, is neglected based on studies that demonstrate strong damping of these waves under weakly collisional conditions for some parameter regimes (Barnes 1966; Foote & Kulsrud 1979); the applicability of this result over the entire range of parameters considered here merits further study, beyond the scope of this work. Here we focus solely on the cascade of Alfvén waves, which are incompressible to lowest order and remain undamped down to the scale of the ion Larmor radius $k_\perp \rho_i \sim 1$ (Schekochihin et al. 2009). If significant turbulent energy exists in the compressive wave modes, then its dissipation and the resulting plasma heating must be accounted for separately; Quataert (1998) discusses this matter in more detail.

Second, the entropy increase required to achieve irreversible thermodynamic heating can only be accomplished through collisions (Howes et al. 2006). Wave-particle interactions damp the electromagnetic fluctuations comprising the turbulence and deposit the associated energy into non-thermal features of the particle distribution function (Howes 2008; Schekochihin et al. 2009). It is assumed that the action of collisions on the non-thermal distributions of a given species lead to heating of that plasma species.

Third, we do not explicitly account for the kinetic energy of the Alfvénic fluctuations at $k_\perp \rho_i < 1$. The restoring force of Alfvén waves is magnetic tension, so the energy in an Alfvén wave is transferred back and forth between magnetic energy and kinetic energy at the wave frequency; for an ensemble of many Alfvén waves, the turbulent fluctuation energy is shared equally between magnetic and kinetic forms. As the turbulence transitions to scales $k_\perp \rho_i \gtrsim 1$, the ions decouple from the turbulence, causing the kinetic energy to become subdominant. It is assumed that the rapid

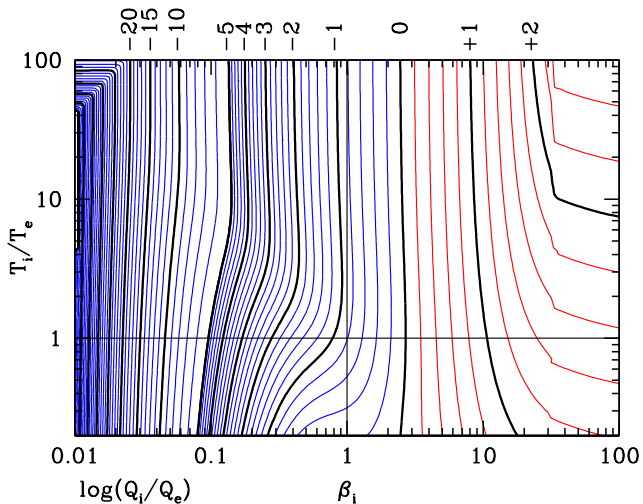


Figure 1. Contour plot of $\log(Q_i/Q_e)$ over the plane $(\beta_i, T_i/T_e)$.

transfer between kinetic and magnetic energy at the Alfvén wave frequency enables both the kinetic and magnetic energy at scales $k_\perp \rho_i < 1$ to be channeled into the dominantly magnetic fluctuations in the kinetic Alfvén wave regime $k_\perp \rho_i \gtrsim 1$. Nonlinear simulations of kinetic turbulence will provide a crucial tool to test this assumption.

3 RESULTS

The key scientific result of this Letter, the calculation of ion-to-electron heating due to the kinetic dissipation of the Alfvénic turbulent cascade, $Q_i/Q_e(\beta_i, T_i/T_e)$, is presented in Fig. 1 as a logarithmic contour plot over the parameter range $0.01 \leq \beta_i \leq 100$ and $0.2 \leq T_i/T_e \leq 100$. The values of the Kolmogorov constants used in this calculation, $C_1 = 1.96$ and $C_2 = 1.09$, are derived from a fit of the cascade model predictions to nonlinear simulations of the transition from the Alfvén to the kinetic Alfvén wave cascade (Howes et al. 2008c).

A salient feature of this result is that the heating rate Q_i/Q_e is primarily a monotonic function of β_i and is nearly independent of T_i/T_e . Quataert (1998) and Quataert & Gruzinov (1999) provide an in-depth discussion of various kinetic damping mechanisms and their dependence on the plasma parameters; here we present only the key concepts necessary to explain this behavior. The ion damping peaks at $k_\perp \rho_i \sim 1$, while the electron damping peaks at $k_\perp \rho_i \gg 1$ unless $T_i/T_e \lesssim m_e/m_i$, a condition unlikely to be relevant to astrophysical plasmas. Therefore, any energy that passes through the peak of the ion damping at $k_\perp \rho_i \sim 1$ will lead to electron heating. The damping onto ions via the Landau resonance is dominated by transit-time damping involving the interaction of the ion’s magnetic moment with the longitudinal magnetic field perturbation δB_\parallel (the magnetic analog of Landau damping by the parallel electric field) (Barnes 1966; Quataert 1998). As the magnetic moment of the ions increases with increasing β_i , so does the strength of the transit-time damping, leading to its dominant role in controlling the ion-to-electron heating Q_i/Q_e . The value of T_i/T_e controls the scale at which the

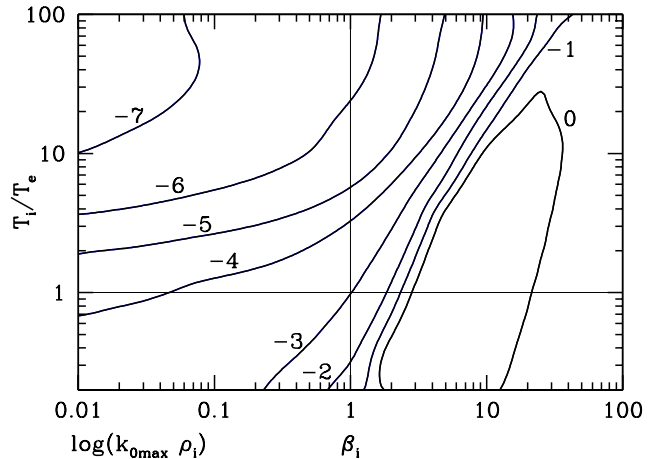


Figure 2. Contour plot of the $\log(k_{0max} \rho_i)$ over the plane $(\beta_i, T_i/T_e)$, a necessary condition for the validity of the heating results presented here.

electron damping occurs, but any energy passing through the range of ion damping will ultimately heat the electrons, leading to little effect on the ratio of total damping Q_i/Q_e .

For values of $T_i/T_e < 0.2$, the slow magnetosonic mode, often called the ion acoustic wave in this limit, is no longer heavily damped. In this case, the energy in this compressive mode may no longer be neglected. Further complicating matters is the fact that the slow wave and Alfvén wave may nonlinearly exchange energy at the scale of the ion Larmor radius (Schekochihin et al. 2009), so the ratio of ion-to-electron heating may no longer be accurately estimated using the present model. Nonlinear kinetic simulations of the turbulence at the scale of the ion Larmor radius will yield valuable guidance to determine the turbulent heating ratio Q_i/Q_e in this regime.

For the gyrokinetic treatment of Q_i/Q_e presented here to be applicable, the turbulence must be dissipated kinetically (via the Landau resonance) before the frequencies rise to the point that the cyclotron resonance alters the dynamics. Since $\omega \propto k_\parallel$ in the gyrokinetic limit, this constraint may be cast as a limitation on the maximum isotropic driving wavenumber k_{0max} (Howes et al. 2008a): for turbulence driven at wavenumbers below k_{0max} , frequencies remain sufficiently low that the cyclotron resonance never contributes and the gyrokinetic treatment is valid. The value of k_{0max} is determined using the following procedure: (1) using the critical balance condition to give $k_\parallel = k_0^{1/3} k_\perp^{2/3} \bar{\omega}^{-1/3} (\epsilon/\epsilon_0)^{1/3}$, the maximum value of k_\parallel/k_0 is determined from the steady-state solution $\epsilon(k_\perp)$; (2) from this point (with fixed k_\perp , β_i , and T_i/T_e), the eigenfrequency using the linear Vlasov-Maxwell dispersion relation is solved as k_\parallel is increased until cyclotron resonant effects lead to $[(\omega_{vm} - \omega_{gk})^2/(\omega_{vm})^2 + (\gamma_{vm} - \gamma_{gk})^2/(\gamma_{vm})^2]^{1/2} > 0.5$; (3) the value of the parallel wavenumber $k_{\parallel max}$ at this point is used to compute the corresponding k_{0max} . A logarithmic contour plot of the value of the normalized maximum isotropic driving wavenumber $k_{0max} \rho_i$ over the $(\beta_i, T_i/T_e)$ plane, presented in Fig. 2, enables verification of the validity of the heating results for a turbulent astrophysical plasma with given values of β_i , T_i/T_e , and k_0 .

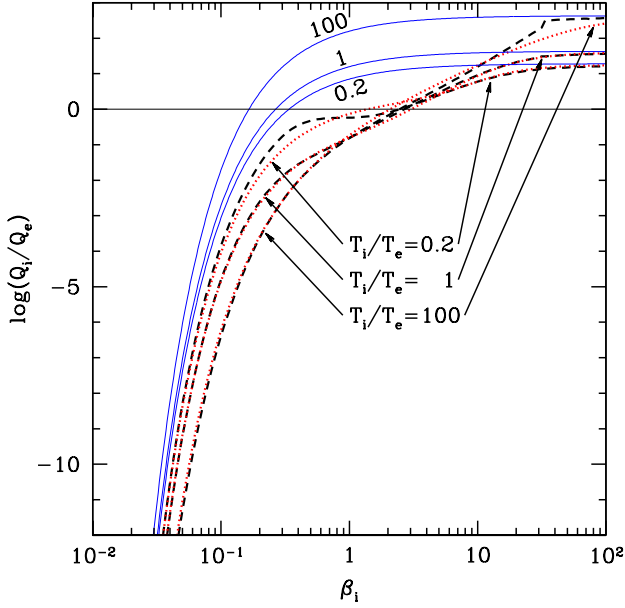


Figure 3. Calculations of $\log(Q_i/Q_e)$ vs. β_i from the cascade model (dashed), the fitted heating prescription eq. (3) (dotted), and eq. (5) of Quataert (1998) (solid).

3.1 Prescription for Turbulent Heating

The second aim of this paper is to provide a prescription for the turbulent heating Q_i/Q_e presented in Fig. 1 that may be used as a sub-grid-scale model for heating in simulations of astrophysical plasma turbulence. The heating prescription, found by fitting the results for Q_i/Q_e in Fig. 1, is given by

$$Q_i/Q_e = c_1 \frac{c_2^2 + \beta_i^p}{c_3^2 + \beta_i^p} \sqrt{\frac{m_i T_i}{m_e T_e}} e^{-1/\beta_i} \quad (3)$$

where $c_1 = 0.92$, $c_2 = 1.6/(T_i/T_e)$, $c_3 = 18 + 5 \log(T_i/T_e)$, and $p = 2 - 0.2 \log(T_i/T_e)$. A slightly better fit for $T_i/T_e < 1$ occurs using the coefficients $c_2 = 1.2/(T_i/T_e)$ and $c_3 = 18$. It is interesting to compare this result to eq. (5) of Quataert (1998), $Q_i/Q_e = [(m_i T_i)/(m_e T_e)]^{1/2} e^{-1/\beta_i}$, which gives the predicted value of ion-to-electron heating by transit time damping in the limit $k_\perp \rho_i < 1$. In Fig. 3, $\log(Q_i/Q_e)$ is plotted for $T_i/T_e = 0.2, 1, 100$ for the three methods: the cascade model calculations (dashed); the heating prescription, eq. (3) (dotted); and eq. (5) of Quataert (1998) (solid). One can see that, although the estimation by Quataert (1998) generally reproduces the behavior of Q_i/Q_e , significant deviations at $\beta_i \sim 1$ are predicted by the cascade model calculations, and the numerical results are much more accurately described by the heating prescription given by eq. (3).

4 DISCUSSION

We consider here the application of eq. (3) to studies of astrophysical accretion disks and to the interstellar medium.

Recent two-fluid studies of the magnetorotational instability (MRI) in accretion disks using shearing box simulations by Sharma et al. (2007) have shown that the development of pressure anisotropies, and their saturation via

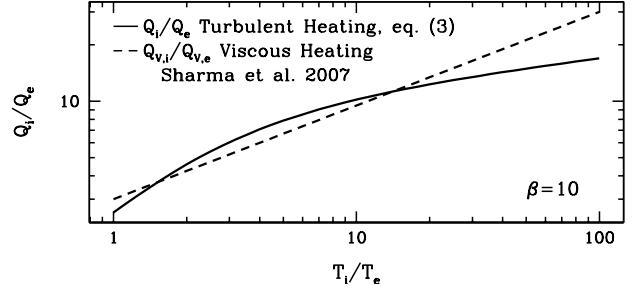


Figure 4. For a $\beta = 10$ plasma, the ion-to-electron heating ratio Q_i/Q_e predicted by eq. (3) (solid) and viscous heating $Q_{V,i}/Q_{V,e}$ arising from anisotropic pressure due to the background shear flow (dashed) from Sharma et al. (2007).

kinetic instabilities, leads to substantial viscous heating, representing a direct conversion of kinetic energy due to differential rotation into heat at large scales. About 50% of the heating in the simulations occurred via this mechanism, with the ratio of ion-to-electron viscous heating given approximately by $Q_{V,i}/Q_{V,e} \approx 3(T_i/T_e)^{1/2}$. Numerical grid-scale dissipation accounted for the remaining 50% of the turbulent energy loss; this loss was not added to the plasma heating because it was unclear which species should receive this energy. By applying the predicted viscous electron heating to Sgr A* using a 1D model for the radiative efficiency, the observed luminosity of $L \approx 10^{36}$ ergs s $^{-1}$ implied an accretion rate of $\dot{M} \sim 10^{-7}$ to $10^{-8} M_\odot$ yr $^{-1}$, well below the estimated Bondi accretion rate (Sharma et al. 2007).

The 50% of the energy lost numerically in their simulations represents the energy transferred to small scales via turbulence, eventually to be damped by collisionless dissipation mechanisms. Here we estimate the partitioning of this heating by species using the turbulent heating model in eq. (3). The MRI drives turbulence on the scale height of the disk (Quataert 1998), so we take the isotropic driving wavenumber to be $k_0 \sim 1/H \sim 10^{-8}$ km for Sgr A*. For an ion temperature of $T_i \sim 10^{12}$ K, and a magnetic field strength of $B \sim 30$ G, the ion Larmor radius is approximately $\rho_i \sim 0.4$ km (Schekochihin et al. 2009). Following Sharma et al. (2007), we take the plasma $\beta_i \sim 10$ and $T_i/T_e \sim 10$, so for a value of $k_0 \rho_i \sim 10^{-7}$, we see from Fig. 2 that we are well below the maximum value $k_{0max} \rho_i \sim 1$ required for the validity of eq. (3).

In Fig. 4, we compare the turbulent heating ratio Q_i/Q_e from eq. (3) for $\beta = \beta_i(1 + T_e/T_i) = 10$ to the viscous heating $Q_{V,i}/Q_{V,e}$ from Sharma et al. (2007). The quite surprising result is that, for a $\beta = 10$ plasma, the two distinct mechanisms yield a heating ratio that agrees within a factor of two over the entire range of T_i/T_e . Therefore, adding in the plasma heating due to the damping of the turbulence via kinetic mechanisms does not change the conclusions of Sharma et al. (2007) regarding Sgr A* discussed above.

Note that, in addition to the turbulent heating prescription presented here, a complete sub-grid model for turbulence would also require prescriptions for the sub-grid Reynolds and Maxwell stresses. The relative insensitivity of the measured stresses to the numerical resolution in the simulations of Sharma et al. (2007), however, suggest that the unresolved fluctuations in those simulations do not con-

tribute substantially to the stresses. In this case, the heating prescription alone may be sufficient to determine the fate of the turbulent energy lost numerically.

Turbulence in the interstellar medium can be probed by interpreting the electron density fluctuations as a passive tracer mixed by the ambient turbulence. Interstellar scintillation measurements of these density fluctuations in the diffuse ISM demonstrate an outer scale turbulent wavenumber $k_0 \sim 1/L_0 \sim 10^{-15} \text{ km}$ (Armstrong et al. 1995). Although the ISM is an inhomogeneous plasma characterized by several different phases, it is the warm ionized component of the ISM that responds to the turbulent electromagnetic fluctuations. Taking typical plasma parameters of $n_i = n_e \sim 0.5 \text{ cm}^{-3}$, $T_i \sim T_e \sim 8000 \text{ K}$, and $B \sim 2 \times 10^{-6} \text{ G}$ (Schekochihin et al. 2009), we find $\rho_i \sim 500 \text{ km}$ and $\beta_i \sim 3$. For $\beta_i = 3$ and $T_i/T_e = 1$, the driving wavenumber of $k_0 \rho_i \sim 10^{-12}$ easily satisfies the constraint in Fig. 2. Applying eq. (3), we predict an ion-to-electron heating ratio of $Q_i/Q_e \simeq 1$ due to the dissipation of turbulence in the warm ionized component of the ISM. As seen in Fig. 3, the predicted heating ratio is a relatively strong function of β_i , so for any particular volume of plasma, the heating may vary as β_i varies, but it is clear from the plot that in the range around $\beta_i \sim 3$ neither the ions nor the electrons are dominantly heated. Although there is a substantial cold, weakly ionized component of the ISM, collisional coupling with the warm ions is extremely weak at the dissipative scales, so it is unlikely to affect this prediction significantly.

5 CONCLUSION

This Letter presents a calculation of the ratio of ion to electron heating Q_i/Q_e due to the kinetic dissipation of Alfvénic turbulence in astrophysical plasmas based on the turbulent cascade model of Howes et al. (2008a). The results demonstrate that the heating ratio Q_i/Q_e is primarily a monotonic function of β_i and has weak dependence on T_i/T_e , as seen in Fig. 1. A fit of the cascade model calculations is given by eq. (3), a form suitable for use as a sub-grid-scale model for turbulent heating in simulations of astrophysical plasmas, potentially enabling the determination of observational signatures caused by radiation emitted from the turbulently heated plasma. Future work aims to employ nonlinear kinetic simulations of the dissipation of astrophysical turbulence to test these predictions of the turbulent plasma heating.

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